CLASA A V A

1)	a) $(25-4*6)+2^2$	2p
	$1+4=5$	1p
	b) $5(1+2+3+....+403)$	1p
	$5*403*404:2$	1p
	rezultat 407030	1p
	c) $5+2+10+2+15+2+.....+2015+2$	1p
	$407030 + 2*404$	1p
	Rezultat 407838	1p
2)	a) $7+9+8+3=27$	2p
	b) $A= 999...997983$	2p
	$B= 9*2013+27=18144$	2p
	c) $C=18$	1p
	$u(18^{2017}) = u(8^{2017})=8$	2p
3)	$a^2+a = 239+2^{2017b}$	1p
	$a(a+1) = 239+2^{2017b}$	1p
	$a(a+1)$ nr par	1p
	239 impar, implică 2^{2017b} impar	1p
	deci $2017b=0$, adică $b=0$	1p
	$a(a+1)= 240$	1p
	$240=15*16$	1p
	$a=15$	1p
	final 2017	1p
4)	$\overline{cb} +1\geq 11$	2p
	$2^{11} = 2048$	1p
	$\rightarrow a \leq 2$	1p
	a=1, fals, deci a=2	2p
	$\overline{cb} =10$	1p
	$\overline{3d} =32,$ d=2	1p
	final $\overline{abcd} =2012$	1p

Notă: La fiecare subiect în parte se acordă 1 punct din oficiu

CLASA A VI A

Sub I a) 20+17=37; 2*1≠0 (1p+1p) 2p

b) $\overline{xy} \in \{10, 11, 12, ... ,27\}$ 2p

Deci cardA=18 1p

c) $\overline{xyzt} = 100\overline{xy} + \overline{zt} = 99\overline{xy} + 37$ 1p

$\overline{xyzt} - \overline{mnpq} = 99(\overline{xy} - \overline{mn})$ 1p

dar $(\overline{xy} - \overline{mn}) \in \{1, 2, 3, ... , 17\}$ 1p

final S= 99(1+2+3+...+17) = 15147 1p

Sub II Fie d=(2x+3,2x-7) → d/10 2p

cum 2x+3 e impar, d=5 2p

2x+3=5t→ 2x=5t - 3 , t=2k+1, x=5k+1, k=0,1,2,3,.....,999 1p

x_{1000} = 5*999+1=4996 1p

x_n = 5n-4, n=1,2,3,...., 1000 1p

$x_p+x_q - x_{p+q-1}$= 5p-4+5q-4-(5(p+q-1) - 4) = 1 2p

Sub III m(<COB) = 3m(COD) 2p

m(COD) = 30^0 1p

m(AOB) = 120^0 2p

m(BOD)=60^0 1p

m(BOE) = 60^0 1p

m(AOE) = 180^0 1p

final 1p

Sub IV Δ ABM≡ Δ CNB (CC) 2p

MB=NB 1p

<MBA+ <NBC =90^0 1p

final (tr dr isoscel) 1p

b) Caz I AB<BC

QB ⊥ MD, DA ⊥ MQ, rezultă B ortocentru în tr.MQD 1p

Deci MB ⊥ QD, dar MB ⊥ NB, rezultă NB//DQ 1p

Caz II AB>BC

DM ⊥ BQ, QA ⊥ BD, rezulta M ortocentru tr QDB 1p

deci BM ⊥ QD, dar BM ⊥ NB deci NB//QD 1p

Notă: La fiecare subiect în parte se acordă 1 punct din oficiu

CLASA A VII A

SUB I

a) A={5*0+2, 5*1+2, 5*2+2, ... , 5*403+2 } 1p

card A= 404 2p

b) u(x+1)∈ {3, 8 } (pt orice x din A) 1p

deci x+1 nu e patrat perfect, de unde concluzia 1p

c) Sol 1 (fol inegalitatea mediilor)

$\sqrt{2*7}$ <(2+7)/2=9/2, $\sqrt{12*17}$ <29/2, ... $\sqrt{2012*2017}$< 4029/2 2p

$\sqrt{P}$< (9/2)(29/2).....(4029/2)< (10*30*50*....*4030)/2^{202} = (10^{202}*I_{202})/2^{202} =$5^{202}I_{202}$ 2p

Sol 2

Pt n par (5n+2)(5n+7)< $[5(n+1)]^2$ 2p

Inlocuind n=0, 2, 4, ... , 202, rezulta inegalitatea ceruta 2p

Sub II

a) 1+3+5=9 2p

b) 2017*1009/1009= 2017, nr natural 3p

c) $\frac{1+2+3+\cdots+(2n+1)}{\sqrt{1+3+5+\cdots(2n+1)}}=\frac{(2n+1)(n+1)}{n+1}$ = 2n+1 2p

a= 1+3+5+..... +2n+1=$(n+1)^2$ 2p

n=2016 1p

Sub III

$(2n+1)^3 - (2n - 1)^3 = 24n^2 + 2$ 3p

$= 16n^2 + 4n^2 + 4n + 1 + 4n^2 - 4n + 1$ 3p

$= (4n)^2 + (2n+1)^2 + (2n - 1)^2$. 3p

Sub IV

Fie BC=a, BM=x

BCNM trapez dreptunghic ortodiagonal $\Rightarrow BC^2=BM*CN$ 2p

$\Rightarrow a^2=x*2a, a\neq 0 \Rightarrow a=2x \Rightarrow x=\frac{a}{2}$ 1p

În ΔBMC , m($\sphericalangle B$)= 90° şi $BO\perp CM \Rightarrow BO=\frac{BM*BC}{CM}$ 1p

$CM=\frac{a\sqrt{5}}{2}$ cm, $BO=\frac{a}{\sqrt{5}}$ 2p

$a=2\sqrt{5}$ cm..... 1p

AB= $5\sqrt{5}$ cm, BC= $2\sqrt{5}$ cm..... 1p

$A_{ABCD}=50cm^2$..... 1p

Notă: La fiecare subiect în parte se acordă 1 punct din oficiu

Clasa a VIII a

Sub I

a) $M_a = 7$ 2p

$M_g = 2$ 2p

b) $x = \sqrt{6 + 2\sqrt{5}}$ 2p

$x = \sqrt{5} + 1$ 1p

$y = \sqrt{5} - 1$ 1p

$M_a = \sqrt{5}$, $M_g = 2$ 1p

Sub II

$13^2 = 169$ 2p

$31^2 = 961$ 2p

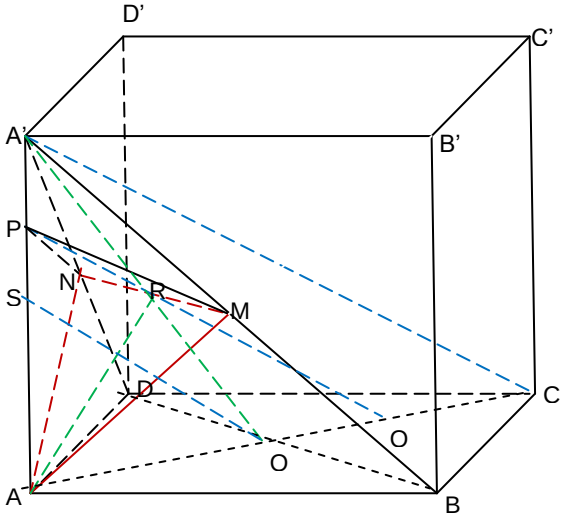
$1000....003^2 = 100...060.....09$ (1p+1p demonstr.) 2p

$3000....001^2 = 900...060.....01$ (1p+1p demonstr.) 2p

final, concluzia 1p

Sub III

In cubul ABCDA'B'C'D' cu latura de 40 cm, notam cu M, N, R mijloacele segmentelor [A'B], [A'D], respectiv [MN]. Fie $P \in [AA']$ asa incat A'P= 10 cm, iar $PR \cap (ABC) = \{Q\}$. Calculati CQ.



SOLUTIA I:

$BC \perp (ABB')$ } $\Rightarrow BC \perp AM \Rightarrow AM \perp BC$ (1)

$AM \subset (ABB')$

Dar $AM \perp A'B$ (2)

$A'B, BC \subset (A'BC)$ (3)

$A'B \cap BC = \{B\}$ (4)

Din (1), (2), (3), (4) $\Rightarrow AM \perp (A'BC) \Rightarrow A'C \subset (A'BC)$

$\Rightarrow AM \perp A'C \Rightarrow A'C \perp AM$ (5)…….. 2p

Analog, $A'C \perp AN$ (6)…….. 1p

$AM, AN \subset (AMN)$ (7)

$AM \cap AN = \{A\}$ (8)

Din (5), (6), (7), (8) $\Rightarrow A'C \perp (AMN)$ (A)…….. 2p

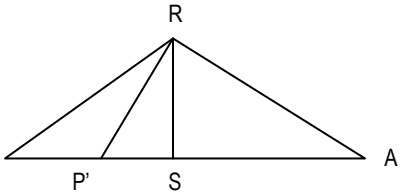
Evident, [PM] ≡ [PN]

Cum R este mijlocul lui [MN] $\Rightarrow PR \perp MN$ (9)…1p

Aratam ca $PR \perp AR$

Fie $P' \in AA'$ a. i. $P'R \perp AR$ si S mijlocul lui [AA']

[A'M] ≡ [AM] } din cazul I.C. $\Rightarrow \Delta AMR \equiv \Delta A'MR \Rightarrow$



[RM] ≡ [RM]

$\Rightarrow$[RA] ≡ [RA'] $\Rightarrow \Delta ARA'$ isoscel $\Rightarrow RS \perp AA'$

In $\Delta ARP'$, $\cos(\prec A)= \frac{AR}{AP'}$ } $\Rightarrow \frac{AR}{AP'}=\frac{AS}{AR} \Rightarrow AP'= \frac{AR\cdot AR}{AS}= \frac{2AR\cdot AR}{AA'}$ (11)

In ΔARS, $\cos(\prec A)=$ }

[MN] linie mijlocie in $\Delta A'BD \Rightarrow MN= \frac{BD}{2}$ } $\Rightarrow \Delta AMN$ echilateral $\Rightarrow AR= \frac{AM\sqrt{3}}{2}= \frac{AB\cdot\sqrt{6}}{4}=10\sqrt{6}$ cm, (12)

AM= AN= $\frac{BD}{2}$

Din (11) si (12) $\Rightarrow AP'= \frac{3}{4}\cdot l = 30$ cm $\Rightarrow P=P' \Rightarrow PR \perp AR$ (13)……. 2p

MN, AR $\subset$ (AMN) (14)

MN $\bigcap$ AR ={ R} (15)

Din (9), (13), (14), (15) $\Rightarrow PR \perp$ (AMN) (B)

Din (A) si (B) $\Rightarrow$ PR $\|$ A'C $\Rightarrow$ PQ $\|$ A'C …… 1p

$\Rightarrow Q \in AC$ si $QC=\frac{AC}{4} \Rightarrow QC= \frac{40\sqrt{2}}{4}=10\sqrt{2}$ cm…….. 1p

SOLUTIA a II-a

Δ A'BD echilateral …… 2p

A', R, O coliniare…….……..... 1p

MN linie mijlocie in Δ A'BD $\Rightarrow$R este mijlocul lui [A'O]…… 1p

Fie S mijlocul lui [AA'] $\Rightarrow$ P este mijlocul lui [A'S]

$\Rightarrow$ PR linie mijlocie in Δ A'SO $\Rightarrow$PR $\|$ SO…….. 1p

SO linie mijlocie in Δ AA'C $\Rightarrow$SO $\|$ A'C……. 1p

$\Rightarrow$PR $\|$ A'C $\Rightarrow$PQ $\|$ A'C……... 1p

$\Rightarrow QC=\frac{AC}{4} \Rightarrow QC= \frac{40\sqrt{2}}{4}=10\sqrt{2}$ cm…… 2p

Sub IV

a) BC//YT, TP=CY, rezulta BCYT trapez isoscel 1p

BC=$a\sqrt{2}$, TY=$2a\sqrt{2}$, TB = $a\sqrt{5}$, h=$3a\sqrt{2}/2$, S= $9a^2/2$ 2p

b) Dacă BD $\perp$ TY, D $\in$ TY, tr BDY e dr. isoscel 2p

obținem m(<YBC)= 45^0 , rezulta m(<TC, BY) = 90^0 1p

c) ICAT trapez → AI∩CT = P, IP/PA= CP/PT=1/2 2p

Dar BQ/QY=CQ/QT = 1/2 , deci P=Q 1p

Notă: La fiecare subiect în parte se acordă 1 punct din oficiu

CLASA a IX-a 3 ore

1. a) $n=2 \Rightarrow x^2-11x-42=0 \Rightarrow \Delta=289$; $x_1=-3$; $x_2=14$ ... (3p)

b) $\Delta=(2n^2+3n-3)^2-4\cdot(6n^2+9n)=(2n^2+3n-3)^2-4\cdot 3\cdot(2n^2+3n)=$

$=(2n^2+3n+3)^2$ ... (2p)

$x_1=2n^2+3n>0$, $x_2=-3<0$ ... (1p)

$S_n=2n^2+3n \Rightarrow a_{n+1}=S_{n+1}-S_n=4n+5 \Rightarrow a_n=4n+1$ ... (1p)

$a_{n+1}-a_n=4$ constantă

$a_1=5$; $a_1=S_1=5 \Rightarrow (a_n)_{n\geq 1}$; $a_1=5$; $r=4$. ... (2p)

2. a) $\frac{x+2}{x^2+3x+4}=-2 \Rightarrow -2x^2-6x-8=x+2 \Rightarrow 2x^2+7x+10=0$

$\Delta=49-80<0 \Rightarrow \nexists x\in\mathbb{R}$ a.î. $f(x)=-2 \Rightarrow -2\notin \text{Im}f$. ... (2p)

$\frac{x+2}{x^2+3x+4}=\frac{1}{2} \Rightarrow x^2+3x+4=2x+4 \Rightarrow x^2+x=0 \Rightarrow x_1=0$; $x_2=-1$ ... (2p)

$\Rightarrow \frac{1}{2}\in \text{Im}f$.

b) $\frac{x+2}{x^2+3x+4}=y$, $y\in\mathbb{R} \Rightarrow yx^2+(3y-1)x+4y-2=0$

$\Delta_y=-7y^2+2y+1$; $\Delta_y\geq 0 \Rightarrow y\in\left[\frac{1-2\sqrt{2}}{7}; \frac{1+2\sqrt{2}}{7}\right]=\text{Im}f$ ... (3p)

$\Rightarrow I\in f(\mathbb{Z})\cap\{0;-1\} \Rightarrow A=\{-1;0\} \Rightarrow 1^2+0^2=1$ ... (2p)

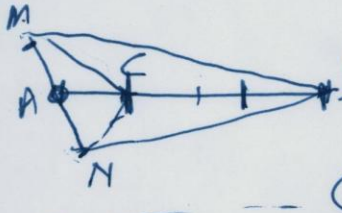
3. Ținând cont de condițiile pentru $a, b, c \Rightarrow$

$\Rightarrow x=a+2b+4-2b-3c+3+3c-a-12=42$ ... (4p)

Analog $y=42$ ... (4p)

$\Rightarrow x=y=42\in\mathbb{N}$ ... (1p)

4.



$\vec{AB}=3\vec{AC}$ ... (1p)

a) $\vec{MC}=\vec{MA}+\vec{AC}=\vec{MA}+\frac{1}{3}\vec{AB}=$

$=\vec{MA}+\frac{1}{3}(\vec{AM}+\vec{MB})=$ ... (2p)

$=-\frac{2}{3}\vec{AM}+\frac{1}{3}\vec{MB}$ ... (2p)

b) $C=$ c.g. al $\triangle BMN$. ... (2p)

$\vec{CM}+\vec{CB}+\vec{CN}=\vec{0}$ ... (2p)

CLASA a IX-a 4 ore

1. a) $a^2+b^2 \geq 2ab$, $a,b>0 \Leftrightarrow (a-b)^2 \geq 0$ (A) — — — — (3p)

b) $\frac{2x^2+1}{x^2+2} > 0$; $\forall x \in \mathbb{R} \Rightarrow \frac{2x^2+1}{x^2+2} + \frac{x^2+2}{2x^2+1} \geq 2$, $\forall x \in \mathbb{R}$ (1) — — (2p)

Dar $\frac{2x^2+1}{x^2+2} + \frac{x^2+2}{2x^2+1} \leq 2$, $\forall x \in \mathbb{R}$ (1) $\Rightarrow \frac{2x^2+1}{x^2+2} + \frac{x^2+2}{2x^2+1} = 2$ — — — (2p)

$\Rightarrow \dots \Rightarrow \frac{2x^2+1}{x^2+2} = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$. — — — (2p)

2. $\left[\frac{4x+1}{3}\right] = \frac{6x-1}{2} \overset{not}{=} k$, $k \in \mathbb{Z} \Rightarrow x = \frac{2k+1}{6}$

$k \leq \frac{4x+1}{3} < k+1 \Rightarrow 3k \leq 4k+1 < 3k+3 \Rightarrow 9k \leq 4k+5 < 9k+9 \Rightarrow k \in \{0;1\}$ — — — (3p)

$\Rightarrow x_1 = \frac{1}{6}$; $x_2 = \frac{1}{2}$ $\Rightarrow q = \frac{1}{2}$ — — (2p)

$\left|\frac{3x}{2} - 1\right| = |x+1| \Rightarrow \dots \Rightarrow x_1 = 4$; $x_2 = 0 \Rightarrow a_1 = 4$ — — — (2p)

$\therefore (a_n)_{n \geq 1}$; $a_1 = 4$; $q = \frac{1}{2}$

$S_{2017} = 4\left(\left(\frac{1}{2}\right)^{2017} - 1\right) \cdot \frac{1}{\frac{1}{2} - 1} = 8\left(1 - \frac{1}{2^{2017}}\right) \in (7;8) \Rightarrow [S_{2017}] = 7$ — (2p)

3. a) Se efectuează operațiile și se obține $(ax-by)^2 \geq 0$ (A), $\forall a,b,x,y \in \mathbb{R}$ (2p)

b) Conform punctului a) pentru $a=b=1$, $x=\sqrt{n^2-n+1}$, $y=\sqrt{n^2+n+1}$ — — (1p)

avem că $\left(\sqrt{n^2-n+1} + \sqrt{n^2+n+1}\right)^2 \leq 2(2n^2+2) \Leftrightarrow$

$\Leftrightarrow \sqrt{n^2+n+1} + \sqrt{n^2+n+1} \leq 2\sqrt{n^2+1}$ — — — (1p)

Cum $2\sqrt{n^2+1} < 2n+1$, $\forall n \in \mathbb{N}^* \Rightarrow \sqrt{n^2-n+1} + \sqrt{n^2+n+1} < 2n+1$, $\forall n \in \mathbb{N}^*$ (1p)

Se arată că $2n \leq \sqrt{n^2-n+1} + \sqrt{n^2+n+1}$, $\forall n \in \mathbb{N}^*$ — — (1p)

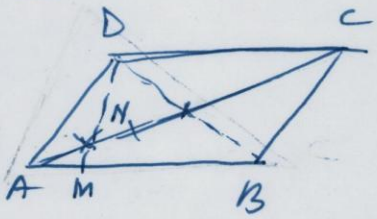
Deci $\left[\sqrt{n^2-n+1} + \sqrt{n^2+n+1}\right] = 2n$, $\forall n \in \mathbb{N}^*$ — — (1p)

Se arată că $n+2 \leq \sqrt{n^2+6n} < n+3$, $\forall n \geq 2$

$\Rightarrow \left[\sqrt{n^2+6n}\right] = n+2$, $\forall n \geq 2$ — — (1p)

Soluțiile ecuației sunt $n \in \{1;2\}$ — — (1p)

4. M, N, D coliniare $\Rightarrow \overrightarrow{DN} \parallel \overrightarrow{DM}$ — — (1p)



$\overrightarrow{DM} = \overrightarrow{DA} + \overrightarrow{AM} = -\overrightarrow{AD} + a \cdot \overrightarrow{AB}$ — — (1p)

$\overrightarrow{DN} = \overrightarrow{DA} + \overrightarrow{AN} = \overrightarrow{DA} + \frac{1}{6}\overrightarrow{AC} =$ — — (2p)

$= -\overrightarrow{AD} + \frac{1}{6}(\overrightarrow{AD} + \overrightarrow{DC}) = -\frac{5}{6}\overrightarrow{AD} + \frac{1}{6}\overrightarrow{AB}$ — — (2p)

$(\overrightarrow{AD}; \overrightarrow{AB})$ - bază — — — — — — (1p)

$\overrightarrow{DN} \parallel \overrightarrow{DM} \Leftrightarrow -1 \cdot \frac{-6}{5} = 6a \Rightarrow a = \frac{1}{5}$ (2p)

CLASA a X-a 3 ore

1. a) $\bar{z} = \frac{-1-i\sqrt{3}}{2} \Rightarrow z+\bar{z} = -1;\ z\bar{z} = 1 \Rightarrow z^2+z+1=0 \Rightarrow z^3 = 1$ — (2p.)

$z^{2020} + \frac{1}{z^{2017}} = z^{2010+1} + \frac{1}{z^{2016+1}} = \epsilon + \frac{1}{\epsilon} = \frac{\epsilon^2+1}{\epsilon} = \frac{-\epsilon}{\epsilon} = -1$ — (3p.)

b) $\varepsilon^3 = 1$; $\varepsilon^2 = \bar{\varepsilon}$

$(a+b\cdot\varepsilon^{2017}+c\varepsilon^2)(a+b\cdot\varepsilon^{2018}+c\varepsilon) = (a+b\cdot\varepsilon+c\varepsilon^2)(a+b\varepsilon^2+c\varepsilon) =$ — (2p.)

$= (a+b\varepsilon+c\varepsilon^2)\cdot(a+b\bar{\varepsilon}+c\bar{\varepsilon}^2) = |a+b\varepsilon+c\varepsilon^2|^2 \geq 0$ — (2p.)

2. a) $9^x+9>0 \ (A),\ \forall x\in\mathbb{R}$; $28-2\cdot 3^x>0 \Rightarrow 3^x<14 \Rightarrow x<\log_3 14$ (1p.)

$\log_3(9^x+9) = \log_3 3^x(28-2\cdot 3^x) \Rightarrow 9^x+9 = 28\cdot 3^x - 2\cdot 9^x \Rightarrow$ (1p.)

$\Rightarrow 3\cdot 9^x - 28\cdot 3^x + 9 = 0$; $3^x \overset{not}{=} y,\ y>0 \Rightarrow 3y^2-28y+9=0$ (1p.)

$\Delta = 16\,49 - 4\cdot 27 = 4\cdot 169 \Rightarrow y_1 = 9;\ y_2 = \frac{1}{3} \Rightarrow x_1 = 2;\ x_2 = -1$ (2p.)

b) $x \leq 0$ (1p.)

$2^x - 4^x = \sqrt{8^x - 16^x} \Rightarrow 2^x - 4^x = \sqrt{4^x(2^x-4^x)}$ — — — (1p.)

$2^x - 4^x = t > 0 \Rightarrow t^2 = 4^x\cdot t \Rightarrow t_1 = 0;\ t_2 = 4^x$ — — (2p.)

Avem $2^x - 4^x = 4^x \Rightarrow 2^x = 4^{x+1} \Rightarrow x = -\frac{1}{2} < 0$. — —

3. a) $\frac{a^6+b^6}{a^4-a^2b^2+b^4} = \frac{(a^2)^3+(b^2)^3}{a^4-a^2b^2+b^4} = \frac{(a^2+b^2)(a^4-a^2b^2+b^4)}{a^4-a^2b^2+b^4} = a^2+b^2$ — (3p.)

b) $\sqrt[3]{4+x} \overset{not}{=} a$; $\sqrt[3]{5-x} = b$.

Avem $a^2+b^2 = 3+ab \Rightarrow a^2-ab+b^2 = 3. \ /^2 \Rightarrow a+b=3$ (3p.)

$a^3+b^3 = 9 \Rightarrow (a+b)(a^2-ab+b^2) = 9$

$S = a+b$; $P = ab$.

Avem: $\begin{cases} S=3 \\ S^2-3P=3 \end{cases} \Rightarrow \begin{matrix} P=2 \\ S=3 \end{matrix} \Rightarrow \begin{cases} a=1 \\ b=2 \end{cases}$ sau $\begin{cases} a=2 \\ b=1 \end{cases}$. — (2p.)

$\Rightarrow x_1 = 5$; $x_2 = -3$. — (1p.)

— — — — — (2p.)

4. a) Prin calcul direct. — (2p.)

b) $a,b,c \in (1;+\infty)$ sau $a,b,c \in (0;1) \Rightarrow \log_a b,\ \log_b a \ldots > 0$

Avem $\prod \log_a(bc) = \prod\left(\log_a b + \log_a c\right) \geq 8\cdot\sqrt{\log_a b\cdot\log_a c}\sqrt{\log_b c\cdot\log_b a}\sqrt{\log_c a\cdot\log_c b} = 8$.

(2p.) (2p.) (1p.)

CLASA a X-a 4 ou

1. I $z^2+1\neq 0$, atunci $8z^4(z^2+1)=z^{10}+1 \Rightarrow 8z^4=\frac{z^{10}+1}{z^2+1} \Rightarrow$ ---- 5p

$\Rightarrow 8z^4=z^8-z^6+z^4-z^2+1 \mid \cdot 8=|z^8-z^6+z^4-z^2+1| \leq 1+1+1+1+1$ (F)

$|z|=1$

II $z^2+1=0 \Rightarrow z^2=-1 \Rightarrow z=\pm i$ se verifică ecuația. } 4p

$\Rightarrow S=\{\pm i\}$

2. a) Prin calcul direct; $a=b=c$ sau $\frac{a}{x}=\frac{b}{y}$. --- 3p

b) $E=\frac{\log_a^2 bc}{4}+\frac{\log_b^2 ac}{4}+\frac{\log_c^2 ab}{4} \geq \dots \geq \frac{(\log_a bc+\log_b ac+\log_c ab)^2}{4+4+4}=$

$=\frac{(\log_a b+\log_a c+\dots)^2}{12} \geq \frac{(2+2+2)^2}{12}=3$ ---- 5p

Se folosește faptul că $\log_a b, \log_b a, \log_a c \dots \in (0;+\infty)$ --- 1p

3. a) $\left(\frac{2017}{2024}\right)^x+7\cdot\left(\frac{1}{2024}\right)^x=1$; $f(x)=\left(\frac{2017}{2024}\right)^x+7\cdot\left(\frac{1}{2024}\right)^x$ } 3p

f s.d. pe $\mathbb{R} \Rightarrow$ f injectivă $\mid \Rightarrow x=1$ sol. unică.

$f(1)=1$

b) f este s.c. (sumă de fct s.c.) $\Rightarrow$ f injectivă. --- 2p

c) $\log_{2017}(\sqrt[3]{x}-3)=y$; $y\in\mathbb{R}$; $\sqrt[3]{x}>3 \Rightarrow x>27$

$\sqrt[3]{x}>-4$

Avem $\sqrt[3]{x}-3=2017^y$ (1)

$\log_{2024}(\sqrt[3]{x}+4)=z \Rightarrow \sqrt[3]{x}+4=2024^z$ (2)

Din (1) și (2) $\Rightarrow 7=2024^z-2017^y \mid \cdot$

Din ecuații avem $2024^y-2017^z=7$ } 4p.

$\Rightarrow 2024^z+2017^z=2024^y+2017^y \Rightarrow f(z)=f(y) \mid \Rightarrow z=y$

f inj

$\Rightarrow 2024^z=7+2017^z \overset{a)}{\Rightarrow} z=1 \Rightarrow x=2021^3>27$

4. a) $C_n^0+C_n^1+C_n^2=171 \Rightarrow 1+n+\frac{n(n-1)}{2}=154 \Rightarrow 2+2n+n(n-1)=308 \Rightarrow$

$\Rightarrow n(n+1)=306 \Rightarrow n(n+1)=17\cdot 18 \Rightarrow n=17$ --- 2p.

b) $T_{k+1}=C_{17}^k\cdot\left(\frac{1}{\sqrt[3]{a^2}}\right)^{17-k}\cdot\left(\sqrt[4]{a^3}\right)^k \Rightarrow \frac{-(34-2k)}{3}+\frac{3k}{4}=\frac{-17}{4}$

$\Rightarrow k=5 \Rightarrow T_6=C_{17}^5\cdot x^{-\frac{17}{4}}$ --- 2p.

c) $T_{k+1}=C_{17}^k\cdot 2^{\frac{-136+17k}{12}}$; $T_{k+2}=C_{17}^{k+1}\cdot 2^{\frac{-119+17k}{12}}$ -- 2p.

$\frac{T_{k+1}}{T_{k+2}}\cdot\frac{C_{17}^{k+1}}{C_{17}^k}=2^{\frac{-136+17k+119-17k}{12}}=2^{-\frac{17}{12}}$ } 3p.

$\frac{15}{2}$

$\sum_{k=0}^{} 2^{-1}=\frac{17}{2}$

CLASA a XI-a 30re

1. a) $A=\begin{pmatrix}-2 & 2\\ 2 & -2\end{pmatrix}$ — — — — (2p).

b) $X=\begin{pmatrix}x & y\\ z & t\end{pmatrix}\in M_2(\mathbb{R})$. — — — (3p).

Din $AX=XA \Rightarrow x=t$; $y=z \Rightarrow X=\begin{pmatrix}x & y\\ y & x\end{pmatrix}$ — — — (1p).

c) $X^3=X^2\cdot X=A\cdot X \;\Big|\overset{b)}{\Rightarrow} X=\begin{pmatrix}x & y\\ y & x\end{pmatrix}$

$X^3=X\cdot X^2=X\cdot A$

$X^2=\begin{pmatrix}x & y\\ y & x\end{pmatrix}\begin{pmatrix}x & y\\ y & x\end{pmatrix}=\begin{pmatrix}x^2+y^2 & 2xy\\ 2xy & x^2+y^2\end{pmatrix} \Rightarrow \begin{cases}x^2+y^2=2017\\ 2xy=2016\end{cases}$ — (1p).

$\Rightarrow \begin{matrix}(x+y)^2=4033\\ (x-y)^2=1\end{matrix} \Rightarrow \begin{cases}x+y=\pm\sqrt{4033}\\ x-y=\pm 1\end{cases} \Rightarrow$ 4 soluții (2p).

2. a) $\Delta=\begin{vmatrix}m & m & 1\\ 1 & m-2 & 1\\ -4 & 2m-1 & 1\end{vmatrix}=m^2-2m+2m-1-4m+4m-8-2m^2+m-m=$ — (2p)

$=-m^2-9\neq 0,\ \forall m\in\mathbb{R} \Rightarrow A, B, C$ sunt — (2p).

vârfurile unui triunghi $\forall m\in\mathbb{R}$.

b) $A_{\Delta ABC}=\frac{1}{2}|\Delta|=\frac{m^2+9}{2}\geq\frac{9}{2},\ \forall m\in\mathbb{R}$. — — (5p).

3. a) $P(2;7)\in G_f \Rightarrow f(2)=7 \Rightarrow \frac{4a+2b+c}{1}=7 \Rightarrow 4a+2b+c=0$ (1p)

$m=\lim\limits_{x\to+\infty}\frac{ax^2+bx+c}{x^2}=a \Rightarrow a=1$ — — (2p).

$n=\lim\limits_{x\to+\infty}\left(\frac{x^2+bx+c}{x}-x\right)=\lim\limits_{x\to+\infty}\frac{bx+c}{x}=b \Rightarrow b=1$ — — (2p).

$a=b=1 \Rightarrow c=-6$. — — — — (1p).

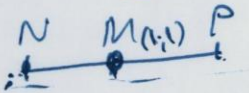
b) $\lim\limits_{x\to+\infty} x\ln\frac{x^2+bx+3}{x^2}\overset{+\infty\cdot 0}{=}\lim\limits_{x\to+\infty}\frac{\ln\left(1+\frac{bx+3}{x^2}\right)}{\frac{bx+3}{x^2}}\cdot\frac{(bx+3)x}{x^2}=b$ — (2p)

$\Rightarrow b=2$. — — (1p).

4. a) $x=1$ asimptotă verticală — — — — (2p).

$y=x$ asimptotă oblică. — — — — (3p).

b) $\begin{cases}x=1\\ y=x\end{cases} \Rightarrow M(1;1)$. — — — (1p).



$M(1;1)$ centru de simetrie $\Leftrightarrow f(1-x)+f(1+x)=2,\ \forall x\in\mathbb{R}\setminus\{1\}$

$1-x+\frac{m}{-x}+1+x+\frac{m}{x}=2$ (A) — — — — (1p).

c) $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}=\ldots=\frac{x^2-2x+1-m}{(x-1)^2}$ — — (1p).

$x^2-2x+1-m=0$ — — — (1p).

$\Delta>0 \Rightarrow m>0$

CLASA a XI-a 4 ore

1. a) $X = \begin{pmatrix} x & y \\ z & t \end{pmatrix} \in M_2(\mathbb{R})$

$A \cdot X = X \cdot A \Rightarrow \dots \quad x = t$ și $y = z \Rightarrow \exists x, y \in \mathbb{R}$ a.î. $X = \begin{pmatrix} x & y \\ y & x \end{pmatrix}$ (2p)

b) $X^4 = X^3 \cdot X = A \cdot X$ $\Big| \Rightarrow A \cdot X = X \cdot A \overset{a)}{\Rightarrow} X = \begin{pmatrix} x & y \\ y & x \end{pmatrix} \in M_2(\mathbb{R})$. (2p)

$X^4 = X \cdot X^3 = X \cdot A$

$X^3 = \begin{pmatrix} \frac{(x+y)^3 + (x-y)^3}{2} & \frac{(x+y)^3 - (x-y)^3}{2} \\ \frac{(x+y)^3 - (x-y)^3}{2} & \frac{(x+y)^3 + (x-y)^3}{2} \end{pmatrix} \Rightarrow \begin{cases} (x+y)^3 + (x-y)^3 = 2 \cdot 2017 \\ (x+y)^3 - (x-y)^3 = 2 \cdot 2016 \end{cases}$ (2p)

$\Rightarrow (x+y)^3 = 4033$ și $(x-y)^3 = 1 \Rightarrow \begin{cases} x+y = \sqrt[3]{4033} \\ x-y = 1 \end{cases} \Rightarrow x = \frac{\sqrt[3]{4033}+1}{2}$

$y = \frac{\sqrt[3]{4033}-1}{2}$ (2p)

$X = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$ (1p)

2. a) Calcul direct. (2p)

b) Ecuația se scrie $(X - 2I_2)^3 = \begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}$; (1p)

$Y \overset{not}{=} X - 2I_2 \Rightarrow Y^3 = \begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix} \Rightarrow (\det Y)^3 = 1 \Rightarrow \det Y = 1$. (1p)

Cu Th. C.-H. $\Rightarrow Y^2 = \operatorname{Tr}Y \cdot Y - I_2 \quad | \cdot Y \Rightarrow Y^3 = (\operatorname{Tr}Y) Y^2 - \operatorname{Tr}Y \cdot I_2 \cdot Y$

$\operatorname{Tr}Y \overset{not}{=} t \in \mathbb{R} \Rightarrow (t^2 - 1) Y = Y^3 + t \cdot I_2 \Rightarrow (t^2-1) \cdot Y = \begin{pmatrix} 4+t & 3 \\ -3 & -2+t \end{pmatrix} \Rightarrow$ (1p)

$\Rightarrow (t^2-1)t = 2t + 2 \Rightarrow (t+1)(t^2 - t - 2) = 0 \Rightarrow t \in \{-1; 2\}$ (2p)

Pt. $t = -1 \Rightarrow$ fals, (1p)

Pt. $t = 2 \Rightarrow Y = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$ (1p)

3. a) f cont. pe $(-\infty, 0]$ și pe $(0; +\infty)$ (1p)

$\ell_s(0) = \lim\limits_{\substack{x \to 0 \\ x < 0}} \frac{e^2(e^{\sqrt{x^2+4}-2} - 1)}{\sqrt{x^2+4} - 2} \cdot \frac{\sqrt{x^2+4} - 2}{x} + a = e^2 \cdot \lim\limits_{\substack{x \to 0 \\ x < 0}} \frac{x^2}{x(\sqrt{x^2+4}+2)} + a =$

$= a \dots$ (2p) $\Big| \Rightarrow a = \frac{1}{2}$. $\dots$ (1p)

$\ell_d(0) = \frac{1}{2} = f(0)$ (1p)

b) $g(x) = f(x) - 2, \ x \in (1; 2) \Rightarrow g(x) = \frac{e^x}{x+1} - 2, \ x \in (1; 2)$ (1p)

$g(1) = \frac{e}{2} - 2 = \frac{e-4}{2} < 0$ (1p) $\Big| \Rightarrow g(1) \cdot g(2) < 0 \Big| \Rightarrow \exists c \in (1; 2)$

$g(2) = \frac{e^2}{3} - 2 = \frac{e^2 - 6}{3} > 0$ (1p) $\quad g$ cont. pe $[1; 2]$ $\quad$ a.î. $g(c) = 2$. (1p)

4. $\frac{x_{n+1}}{x_n} = \frac{1}{(x_n+1)^2 + 1} < 1, \ \forall n \geq 1$, cât că $x_n > 0, \ \forall n \geq 1$ (prin ind.) (3p)

$\Rightarrow (x_n)_{n \geq 1}$ strict descrescător. $\Big| \overset{T.W}{\Rightarrow} (x_n)_n$ conv. $\Rightarrow \exists \ell \in \mathbb{R}$

$(x_n)_n$ mărginș. a.î. $\lim\limits_{n \to +\infty} x_n = \ell$. (2p)

Avem $\ell = \frac{1}{(\ell+1)^2 + 1} \Rightarrow \ell(\ell+1)^2 = 0 \Big| \Rightarrow \ell = 0 \Rightarrow \lim\limits_{n \to \infty} x_n = 0$ (2p)

$\ell \geq 0$

$n^k \cdot x_n = \left(n \sqrt[k]{x_n}\right)^k = \left(\frac{n}{\left(\frac{1}{x_n}\right)^{\frac{1}{k}}}\right)^k$. Avem $\frac{n+1-n}{\left(\frac{1}{x_{n+1}}\right)^{\frac{1}{k}} - \left(\frac{1}{x_n}\right)^{\frac{1}{k}}} = \frac{x_n^{\frac{1}{k}}}{\left|\left(\frac{x_n+1}{x_n}\right)^{\frac{1}{k}} - 1\right|} \to 0$

Din Lema Stolz-Cesaro $\Rightarrow \lim\limits_{n \to +\infty} x_n \cdot n^k = 0$ (2p)

CLASA a XII-a 3 ore

1. a) $x*y = (x-1)(y-1)+1$, $\forall x,y \in \mathbb{R}$ — 2p.

$(x*y)*z = x*(y*z)$, $\forall x,y,z \in \mathbb{R}$.

$(x*y)*z = \dots = (x-1)(y-1)(z-1)+1$; $x*(y*z) = (x-1)(y-1)(z-1)+1$ — 3p.

b) $(x-1)^3+1 = x = 1 \Rightarrow (x-1)^3 = x-1 \Rightarrow x_1 = 1$; $x_2 = 0$; $x_3 = 2$ — 1p.

c) $e = 2$ elementul neutru — 1p.

x simetrizabil $\Leftrightarrow \exists\, x' \in \mathbb{R}$ a.î. $x*x' = x'*x = e$ — 1p.

se obține $x' = \frac{x}{x-1}$, $\forall x \in \mathbb{R} \setminus \{1\}$. — 1p.

$x = 2017 \Rightarrow x' = \frac{2017}{2016}$

$x = 2 \Rightarrow x' = \frac{2}{1}$

Avem $\frac{2017}{2016} * \frac{2016}{2015} * \dots * \frac{4}{3} * \frac{3}{2} * \frac{2}{1} * 2 * 3 * \dots * 2017 = \dots = 2$. — 4p.

2. a) f continuă pe $(-\infty; -1)$ și pe $[-1; +\infty)$ — 3p.

f cont în $x = -1 \Leftrightarrow f(-1-0) = f(-1+0) = f(-1)$

$\Rightarrow a-1 = 4 \Rightarrow a = 5$

f cont pe $\mathbb{R}$ pt $a = 5 \Rightarrow f$ ad primitive pt $a = 5$

b) $g(x) = x^{2017} \cdot \sqrt{3x^2+1}$, $x \in [-1; 1]$. — 3p.

g este impară $\Rightarrow \int_{-1}^{1} g(x)\,dx = 0$

c) f crescătoare pe $[m; m+1]$, $f(x) > 0$, $\forall x \in [m, m+1]$ — 3p.

$A = \int_m^{m+1} (3x^2+1)\,dx = \dots = 3\left(m+\frac{1}{2}\right)^2 + \frac{15}{12}$; $m = -\frac{1}{2}$. — 3p.

3. a) $f = (2x+1)^{2017} \cdot (x-1)^{2017} = (2x+1)^{2017}$ — 3p.

b) $T_{k+1} = C_{2017}^{k} \cdot 2^{2017-k} \cdot x^{2017-k}$; $T_{\ell+1} = C_{2017}^{\ell} \cdot x^{2017-\ell} \cdot (-1)^{\ell}$

$2017-k+2017-\ell = 4033 \Rightarrow k+\ell = 1$

$k = 0$; $\ell = 1 \Rightarrow C_{2017}^{0} 2^{2017} \cdot C_{2017}^{1} \cdot (-1) \cdot 2^{2016}$

$k = 1$; $\ell = 0 \Rightarrow C_{2017}^{1} \cdot 2^{2016} \cdot 2^{2017} \cdot (-1)^{0}$

$\Rightarrow a_{4033} = 0$

c) $x_1 + x_2 + \dots + x_{4034} = -\frac{a_{4033}}{a_{4034}} = 0$ — 3p.

4. a) Fie F o primitivă a lui $f \Rightarrow F$ derivabilă pe $\left(\frac{1}{2}; +\infty\right)$ — 3p.

$F'(x) = f(x)$, $\forall x \in \left(\frac{1}{2}; +\infty\right)$

$F'(x) = f(x) \Rightarrow F''(x) = f'(x) = \frac{1}{\sqrt{2x-1}} > 0$, $\forall x > \frac{1}{2} \Rightarrow F$ convexă

b) $F(x) = \int \sqrt{2x-1}\,dx = \dots = \frac{1}{3}(\sqrt{2x-1})^3 + C$ — 3p.

$\lim_{x \to \infty} F(x) = \frac{1}{3} \Rightarrow C = 0 \Rightarrow F(x) = \frac{1}{3}(\sqrt{2x-1})^3$

c) $\int_1^5 f(x) F^{2017}(x)\,dx = \frac{F^{2018}(x)}{2018}\Big|_1^5 = \frac{3^{6054}-1}{6054}$. — 3p.

CLASA a XII-a 4 ore

1. a) $f_2 = f_1 \circ f_1 \Rightarrow f_2(x) = \frac{x}{\sqrt{1+2x^2}} \Rightarrow I_2 = \int_0^2 f_2(x)\,dx =$ (2p.)

$= \int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx = 1.$ — — — (2p.)

b) $f_n = \frac{x}{\sqrt{1+nx^2}}$ (se dem. prin inductie).

$I_n = \int_0^n \frac{x}{\sqrt{1+nx^2}}\,dx = \frac{1}{n}\left(\sqrt{1+n^3}-1\right)$ (5p.)

$\lim_{n\to\infty} \frac{I_n}{\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}\left(\sqrt{1+\frac{1}{n^3}} - \frac{1}{\sqrt{n}\sqrt{n}}\right)}{n\sqrt{n}} = 1.$

2. a) $f:(x^2-1) \Rightarrow \exists\, q, r \in \mathbb{R}[X]$ a.i. $f = (x^2-1)q + r$, $\operatorname{grad} r < 2$ — (1p.)

$r = aX+b \in \mathbb{R}[X]$

$f(1) = r(1)$; $f(1) = 3^{2017}+1 \Rightarrow a+b = 3^{2017}+1$

$f(-1) = r(-1)$; $f(-1) = 3^{2017}+1 \Rightarrow -a+b = 3^{2017}+1$ $\Rightarrow b = 3^{2017}+1,\ a=0$ (2p.)

$\Rightarrow r = 3^{2017}+1$

b) $a_0 + a_2 + \dots + a_{4034} = \frac{f(1)+f(-1)}{2} = 3^{2017}+1$ — — — — (2p.)

c) $(x^2+x+1)^{2017} = \sum_{k=0}^{2017} (x^2+x)^{2017-k} \cdot C_{2017}^k$ — — — (1p.)

$T_{q+1} = C_{2017-k}^q \cdot (x^2)^{2017-k-q} \cdot x^q = C_{2017-k}^q \cdot x^{4034-2k-q}$ — — (1p.)

$(x^2-x+1)^{2017} = \sum_{k=0}^{2017} (x^2-x)^{2017-k} \cdot$

$T_{q+1} = C_{2017-k}^q \cdot (x^2)^{2017-k-q} \cdot (-1)^q \cdot x^q = C_{2017-k}^q (-1)^q \cdot x^{4034-2k-q}$ — (1p.)

$4034-2k-q = 2017 \Rightarrow 2k+q = 2017 \Rightarrow q$ - impar

$a_{2017} = C_{2017}^k \cdot C_{2017-k}^q - C_{2017-k}^q \cdot C_{2017}^k = 0.$ — — — (1p.)

3. a) $x * y = (x-7)(y-7)+7$

$x-7 = \frac{2}{7} \Rightarrow x = \frac{51}{7} \in \mathbb{Q}\setminus\mathbb{Z}$ $\Rightarrow \frac{51}{7} * \frac{2}{2} = \frac{2}{7}\cdot\frac{2}{2}+7 = 8 \in \mathbb{N}$ — (3p.)

$y-7 = \frac{2}{2} \Rightarrow y = \frac{21}{2} \in \mathbb{Q}\setminus\mathbb{Z}$

b) $f'(x) = \frac{1}{x-7} > 0 \Rightarrow f$ s.c. $\Rightarrow f$ inj $\Big| \Rightarrow f$ bij — — (2p.)

$\operatorname{Im} f = \mathbb{R} \Rightarrow f$ surj

$f(x*y) = f((x-7)(y-7)+7) = \ln((x-7)(y-7)) = \ln(x-7)+\ln(y-7) =$

$= f(x)+f(y) \Rightarrow f$ morfism (1p.)

c) $a = 9 * 10 * \dots * 2021 \Rightarrow f(a) = f(9)+f(10)+\dots+f(2021) =$ — — — (3p.)

$= \ln 2\cdot 3 \cdots 2014 \Rightarrow a = 2014!+7$

4. a) G primitiva a lui $g \Leftrightarrow$ 1) G derivabila pe $(0;+\infty)$ } (3p.)

2) $G'(x) = g(x),\ \forall x \in (0,\infty)$

$G'(x) = \dots = g(x),\ \forall x > 0$

b) f cont pe $[1;e]$, $f(x) > 0,\ \forall x \in [1,e] \Rightarrow A = \int_1^e f(x)\,dx$

$A = \int_1^e \left(\frac{1}{x(2016+\ln x)} - \frac{1}{x(2017+\ln x)}\right)dx \overset{a)}{=} \ln(2016+\ln x)\Big|_1^e - \ln(2017+\ln x)\Big|_1^e =$ (4p.)

$= \ln\frac{2017}{2016} - \ln\frac{2018}{2017} = \ln\frac{2017^2}{2016\cdot 2018}$

c) F primitiva a lui $f \Rightarrow F'(x) = f(x) > 0,\ \forall x \geq 1 \Rightarrow F$ s.c. — (2p.)