BAREM, CLASA A III A

SUBIECTUL I: a=120; b=360 ; c=240 (2x3p) 6p
ordinea a;c;b 1p
S=720; câturile sunt 360; 240; 120 (4x0,5p) 2p
SUBIECTUL II: a) 12:2=6 lei; 18:2=9 caiete (2p+1p) 3p
b) 115-15=100g cântăresc bomboanele din cutie. (0,5p); 6v+7c=100 (0.5p) 1p
Pt ca nr de bomboane să fie max, trebuie să fie mai puține cele de 7g. 1p
c=1, 6v=100-7=93; nu convine (0,5p)
c=2, 6v=100-14=86; nu convine (0,5p)
c=3, 6v=100-21=79; nu convine (0,5p) 1,5p
c=4, 6v=100-28=72; v=12 1,5p
Nr max 4+12=16 bomboane 1p
SUBIECTUL III: 2a=b+c (1p) a=120:3=40 (3p) 4p
c=b+a (1p) c= 120:2=60 (3p) b=20 (1p) 5p
SUBIECTUL IV: 1, 4, 7, 1+4=5, 1+7=8, 4+7=11, 1+4+7=12, 4-1=3, 7-1=6, 7-4=3(se repetă), 7+1-4=4 (se repetă), 7+4-1=10, 7-1-4=2. Se acordă câte 0,5p pentru fiecare caz analizat (total 6,5p), plus 2p bonus dacă le analizează pe toate 13 8,5p
11 cantități diferite de cireșe. 0,5p

NOTĂ: Se acordă 1p din oficiu la fiecare subiect!

BAREM CLASA A IV A

SUBIECTUL I: a) 56 (1p+1p+1p) 3p
b) (11+176):17=11 (1p+1p+1p) 3p
c) 3x1+1+3x2+1+3x3+1+...+3x33+1=1716 (1p+2p) 3p
SUBIECTUL II: a) (a+4)x4+4x4x4=96 1p
(a+4)x4=32 2p
a=4 2p
b) [...]=4 (1p) ; (...):(...)=100 (1p) ; b-1015=1000 (1p); b=2015 (1p) 4p
SUBIECTUL III: a=6c; b=3d (2p); c+d=40 (1p); 6c+3d=195 (1p) 4p
3c=75 (2p), c=25, deci a (primul nr) este 150 (1p) 3p
d=15, adică b (al 2-lea nr) este 45 2p
SUBIECTUL IV:
a) 0,2,4,6,8,20,24,26,28,40,42,46,48,60,62,64,68,80,82,84,86 (0,1p/nr) 2,1p
S=900 0,9p
b) cu 1 sau 2 cifre avem 20 numere
cu 3 cifre $\overline{abc}$, a are 4 posib, b are tot 4 pos, iar c are 3 posib, avem 4x4x3=48nr 2p
cu 4 cifre $\overline{abcd}$, a-4pos; b-4 pos; c-3 pos; d-2pos, sunt 4x4x3x2=96 nr 1p
cu 5 cifre $\overline{abcde}$, a-4pos, b-4pos, c-3 pos, d-2pos, e-1 pos, deci 4x4x3x2x1=96nr 1p
mai mult de 5 cifre nu avem nr *paritare* 0,5p
total 20+48+96+96=260 numere *paritare* 1p
260 e număr *paritar* 0,5p

BAREM CLASA A V A

SUBIECTUL I: $(3x-3,25)\times 4:25=966,68$ 3p

$(...)\times 4=24167$ 2p

$(...)=6041,75$ 2p

$3x=6045$, $x=2015$ (1p+1p) 2p

SUBIECTUL II: $(1550-350):2=600$ km (în localități) 4p

$600+350=950$ km (în afara loc) 1p

$9,5x6,2+6x7,8=105,7$ litri (1,5p+1,5p+1p) 4p

SUBIECTUL III: $25^{2015}=5^{4030}$ (0,5p); $5^{4030}=5^4x5^{4026}$ (1p); $5^4=625$ (0,5p) 2p

$625=400+225=20^2+15^2$ 2p

$625=576+49=24^2+7^2$ 3p

$5^{4030}=(20x5^{2013})^2+(15x5^{2013})^2$ 1p

$5^{4030}=(24x5^{2013})^2+(7x5^{2013})^2$ 1p

SUBIECTUL IV: a) 35,41,47 1p

b) termenii sunt de forma $6k+5$ (sau $6t-1$) 1p

$2015=6x335+5$ (1p), deci 2015 este al 336-lea termen (1p) 2p

c) $B=\{a,b\}$; $(a+b)/2=6k+5$; $(6p+5+6t+5)/2=6k+5$, rezultă $3(p+t)=6k$, deci p+t=par 1p

$p=1\rightarrow t\in\{3,5,7,...,335\}$, deci 167 de submulțimi

$p=3\rightarrow t\in\{5,7,...,335\}$ deci 166 de submulțimi

.....

p=333, t=335, adică o submulțime 1,5p

$p=0\rightarrow t\in\{2,4,6,...,334\}$, deci 167 de submulțimi

$p=2\rightarrow t\in\{4,6,...,334\}$, deci 166 de submulțimi

.....

p=332, t=334, adică o submulțime 1,5p

total: $2(1+2+3+...+167)=167x168=28056$ 1p

CLASA A VI A

SUBIECTUL I : a) $x=5t$, $y=7t$ (1p), raport=5/4 (3p) 4p

b) $6(4x-y)=5(6x-2y)$; (1p); $24x-6y=30x-10y$ (1p); $4y=6x$ (1p); $x/y=2/3$ (1p) 4p

$2/3=0,(6)$, deci cifra cerută e 6 1p

SUBIECTUL II: Def. probabilității 0.5p

a) nr cazuri fav: 44 $(1^2, 2^2, 3^2,....,44^2)$ 2p

p=44/2015 0,5p

b) nr div cu 25 sunt 80 (1p); nediv cu 25: 2015-80=1935 (0,5p) 1,5p

p=1935/2015=387/403 0,5p

c) $\overline{abc}$, $\{a,b,c\}=\{1,3,8\}$ avem 6 numere 1p

$\{a,b,c\}=\{1,4,6\}$ 6 nr 0,5p

$\{a,b,c\}=\{2,2,6\}$ 3 nr 1p

$\{a,b,c\}=\{2,3,4\}$ 6 nr 1p

p=21/2015 0,5p

SUBIECTUL III: $a^2+a=a(a+1)$=nr par (2p); la fel b^2+b (1p) 3p

deci $c+3/(c+1)$ este nr natural (1p), de unde c=0 sau c=1 (2p) 3p

c=1 nu e posibil (nu avem soluții) 1p

c=0, obținem a=0 și b=2 2p

SUBIECTUL IV: a) $A=90^0$, $B=75^0$, $C=15^0$ 3p

b) figura 1p

$m(<MAP)=60^0$ (0.5p); $m(<MPA)=60^0$ (1p), deci tr AMP e echilateral (0,5p) 2p

c) tr APB e isoscel (0,5p), $PQ\perp AB\rightarrow PQ$ mediatoare (0,5p), deci AQ=BQ (0,5p) 1,5p

tr AQC isoscel (0,5p) $\rightarrow$AQ=CQ (0,5p), de unde CQ=BQ (0,5p) 1,5p

BAREM CLASA A VII A

SUBIECTUL I: $|x-2015|=y$ 1p

$|y-2014|=1 \rightarrow y\in\{2015,2013\}$ 4p

$|x-2015|=2015$, obținem x=0 sau x=4030 2p

$|x-2015|=2013$, obținem x=2 sau x=4028 2p

SUBIECTUL II: a) $(\sqrt{6}+2)^2=10+4\sqrt{6}$; $2\sqrt{5}^2=20$; $(\sqrt{6}-2)^2=10-4\sqrt{6}$ 3p

$10-4\sqrt{6}+10+4\sqrt{6}=20$ 2p

deci triunghiul e dreptunghic cu ipotenuza de $2\sqrt{5}$ 1p

b) mediana = $\sqrt{5}$ 1p

înălțimea este $\sqrt{5}/5$ 2p

SUBIECTUL III: a) $a^2-b^2=2015\rightarrow(a-b)(a+b)=2015$ 1p

$2015=5x13x31$ 1p

$a-b<a+b\rightarrow$ a-b=1, a+b=2015, a=1008, b=1007 1p

a-b=5, a+b=403, a=204, b=199 1p

a-b=13, a+b=155, a=84, b=71 1p

a-b=31, a+b=65, a=48, b=17 1p

b) $a^3-b^3=2015 \rightarrow (a-b)(a^2+ab+b^2)=2015$ 1p

a-b=5, obținem a=14, b=9 2p

SUBIECTUL IV: figura 1p

Fie F mijl (BD) (1p) → MF//AD, MF=AD/2 (1p) 2p

→DE//MF și DE=MF/2, deci DE e linie mijlocie în tr NMF 3p

adică E este mijlocul (MN) 1p

D este mijl. (FN) → ND=BC/4, rezultă NC=BC/4, adică N e mijl. (DC) 2p

BAREM CLASA A VIII A

SUBIECTUL I:

1	2	3	4	5	6
84	4	16π	1/2	$2\sqrt{6}$	14

SUBIECTUL II: 1) Fig (4p) notație (1p)

2) 264 (după scumpire) 3p; 237,6 preț final (2p)

3) def med geom (1p) calcul, $m_g=3$ (4p)

4) a) cond. f(a)=a (2p) a=1 (2p), A(1;1) (1p)

b) reprez grafic (2p) intersecție Ox (2p) tangenta=1/3 (1p)

5) $A=(1+2\sqrt{3})+(5-2\sqrt{3})=6\in\mathbf{N}$ (2p+2p+1p)

SUBIECTUL III: 1) a) P=2400m (2p) sârmă= 12000m (3p)

b) $S=360000m^2$ (1p) L=900m (1p) diag=$100\sqrt{97}$ (1p) sin=36/385 (2p)

c) S=36ha (2p) 115200 lei (3p)

2) a) formula arie (1p) $S=6300\ cm^2$ (4p)

b) $V_{acv}=378000\ cm^3$, (1p), $V_{pietriș}=31500\ cm^3$ (1p), $V_{pești}=37800\ cm^3$ (1p), $V_{apă}=308700\ cm^3$ (1p), final 308,7 litri (Ip)

c) $S_l=19200\ cm^2$ (1p) $S_{sticlă}=25500cm^2$ (1p) $=2,55\ m^2$ (1p), cost sticlă 204 lei (0,5p), manoperă 61,2 lei (1p) cost total 265,2 lei (0,5p).

SUBIECTUL IV: figura (0,5p), $A^lN=9\sqrt{3}$ (0,5p) PN⊥MN, PN⊥BC→PN⊥(BCN) (5p), deci e suf. să det P astfel încât PN⊥BN (2p). Fie AP=x, → $A^lP=18-x$

$PN^2=567-36x+x^2$ (3p), $BN^2=351$ (2p) $PB^2=432+x^2$ (2p)

$PB^2=PN^2+NB^2$ (2p) AP=13,5 (3p)

CLASA a IX-a M1

1) a) $a, b, c \Rightarrow b^2 = ac$
$\Delta = b^2 - 4ac \Rightarrow \Delta = -3ac < 0 \Big| \Rightarrow f(x) > 0, \forall x \in \mathbb{R}$, $a > 0$.

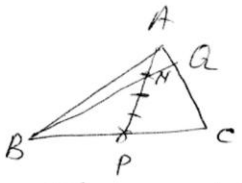
b) $f(1) = 14 \Rightarrow a+b+c = 14$
$b^2 = ac$ și $a = 4c \Rightarrow b^2 = 4c^2 \Rightarrow b = 2c \Big| \Rightarrow 7c = 14 \Rightarrow c = 2$.
$\Rightarrow a = 8$, $b = 4$.

2) a) Prin calcul direct,

b) $x\sqrt{y} = \sqrt{x} \cdot \sqrt{xy} \le \frac{x+xy}{2}$ și analogele.
$\Rightarrow x\sqrt{y} + \dots \le \frac{x+xy+\dots}{2} = \frac{4+xy+yz+zt+tx}{2} \le$
$= \frac{4+(x+z)(y+t)}{2} \le \frac{4+(x+y+z+t)^2}{2} = \frac{4+4}{2} = 4$

3. a) $\overrightarrow{OA} = 2\vec{i} + 3\vec{j}$

b) $M(a;b)$, $a, b \in \mathbb{R}$; $(a-2)\vec{i} + (b-3)\cdot\vec{j} = 3(2\vec{i} - 3\vec{j}) \Rightarrow$
$\Rightarrow \begin{cases} a-2 = 6 \Rightarrow a = 8 \\ b-3 = -6 \Rightarrow b = -3 \end{cases} \Rightarrow M(8; -3)$



b) B, N, Q coliniare $\Rightarrow \overrightarrow{BN} \parallel \overrightarrow{BQ}$

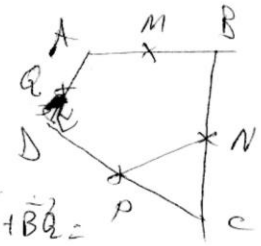
$\overrightarrow{BN} = \overrightarrow{BA} + \overrightarrow{AN} = \overrightarrow{BA} + \frac{1}{4}\overrightarrow{AP} = \frac{7}{8}\overrightarrow{BA} + \frac{1}{8}\overrightarrow{AC}$

$\frac{AQ}{QC} = \alpha$, $\alpha \in \mathbb{R}_+^* \Rightarrow \overrightarrow{BQ} = \frac{1}{\alpha+1}\overrightarrow{BA} + \frac{\alpha}{\alpha+1}\overrightarrow{BC} = \frac{1}{\alpha+1}\overrightarrow{BA} + \frac{\alpha}{\alpha+1}(\overrightarrow{BA}+\overrightarrow{AC})$

$\Rightarrow \overrightarrow{BQ} = \frac{1}{\alpha+1}\overrightarrow{BA} + \frac{\alpha}{\alpha+1}\overrightarrow{BA} + \frac{\alpha}{\alpha+1}\overrightarrow{AC} = \overrightarrow{BA} + \frac{\alpha}{\alpha+1}\overrightarrow{AC}$

$\overrightarrow{BN} \parallel \overrightarrow{BQ} \Rightarrow \frac{7}{8} = \frac{1}{8} \cdot \frac{\alpha+1}{\alpha} \Rightarrow \alpha = \frac{1}{6} \Rightarrow \overrightarrow{AQ} = \frac{1}{6}\overrightarrow{QC}$

4.



a) P mijloc $[DC]$
N mijloc $[BC]$ $\Big| \Rightarrow PN \parallel DB$, $PN = \frac{1}{2}DB$
$\Rightarrow \overrightarrow{PN} = \frac{1}{2}\overrightarrow{DB}$.

b) $\overrightarrow{MP} + \overrightarrow{NQ} = \overrightarrow{MB} + \overrightarrow{BN} + \overrightarrow{NP} + \overrightarrow{NB} + \overrightarrow{BQ} =$
$= \overrightarrow{MQ} + \overrightarrow{NP} = \frac{1}{2}\overrightarrow{BD} + \frac{1}{2}\overrightarrow{BD} = \overrightarrow{BD}$

CLASA a X-a M2.

1. a) $(x+1)^2 > 0 \Rightarrow x \in \mathbb{R} \setminus \{-1\}$

$\log_3 (x+1)^2 = \log_3 |x+1| \Rightarrow (x+1)^2 = |x+1| \Rightarrow x = 0 ; x = -1$ soluții

b) $x^2 - 1 \geq 0 \Rightarrow x \in (-\infty; -1] \cup [1; +\infty)$

$\left(\sqrt{4-\sqrt{15}}\right)^{x+\sqrt{x^2-1}} \overset{not}{=} y, \ y > 0 \Rightarrow y + \frac{1}{y} = 8 \Rightarrow y^2 - 8y + 1 = 0$

$\Rightarrow y_1 = 4 - \sqrt{15}; \ y_2 = 4 + \sqrt{15}$

$\left(\sqrt{4-\sqrt{15}}\right)^{x+\sqrt{x^2-1}} = 4 - \sqrt{15} \Rightarrow x + \sqrt{x^2-1} = 2 \Rightarrow \sqrt{x^2-1} = 2 - x, \ 2 - x \leq 0$

$\Rightarrow x = \frac{5}{4}$ soluții

$\left(\sqrt{4+\sqrt{15}}\right)^{x+\sqrt{x^2-1}} = 4 + \sqrt{15} \Rightarrow x + \sqrt{x^2-1} = -2 \Rightarrow \sqrt{x^2-1} = -x - 2, \ -x - 2 \leq 0$

$\Rightarrow x = -\frac{5}{4}$ soluții

2. a) $2x - 1 \geq 2; \ 2x - 1 \in \mathbb{N} \Rightarrow 2x \in \{3; 4; 5; \dots\}$.

$(2x-1)\cdot(2x-2) = 3 \Rightarrow (2x-1)(2x-2) = 2 \cdot 3 \Rightarrow 2x - 1 = 3 \Rightarrow x = 2$ soluții

b) $2^{n-1} + 2^n + 2^{n+1} = 448 \Rightarrow 2^{n-1} = 64 \Rightarrow n = 7$

$T_{k+1} = C_7^k a^{7-k} \cdot 5^k; \ C_{n-1}^3 + C_n^3 + C_{n+1}^3 =$ coef. lui x^4

$\Rightarrow C_6^3 + C_7^3 + C_8^3 = 20 + 35 + 56 = 111 =$ coef. lui x^4

3. a) Ecuația divine $z^4 + 24z^2 - 25 = 0 \Rightarrow z^2 = 1; \ z^2 = -25 \Rightarrow z = \pm 1; z = \pm 5i$

b) Suma oricăror patru puteri consecutive ale lui i este nulă. Cum avem 4031 de puteri, vor rămâne ultimele trei $\Rightarrow A = -1$

$B = ((1+i)^2)^{1007} \cdot (1+i) - ((1-i)^2)^{1007} \cdot (1-i) = 2^{1007} \cdot (1-i) - 2^{1007} \cdot (1+i) = -2^{1007} \cdot i$

4. Fie $x_1, x_2 \in \mathbb{R} \setminus \{2015\}$ a. î. $f(x_1) = f(x_2) \Rightarrow \frac{1+2015x_1}{x_1 - 2015} = \frac{1+2015x_2}{x_2 - 2015} \Rightarrow$

$\Rightarrow x_1 = x_2 \Rightarrow f$ injectivă.

f surjectivă $\Leftrightarrow \forall y \in \mathbb{R} \setminus \{2015\}, \exists x \in \mathbb{R} \setminus \{2015\}$ a. î. $y = f(x)$

$y = f(x) \Rightarrow y = \frac{1+2015x}{x-2015} \Rightarrow x = \frac{1+2015y}{y-2015} \in \mathbb{R} \setminus \{2015\} \Rightarrow f$ surj

$f^{-1}: \mathbb{R} \setminus \{2015\} \to \mathbb{R} \setminus \{2015\}; \ f^{-1}(x) = \frac{1+2015x}{x-2015} = f(x), \ \forall x \in \mathbb{R} \setminus \{2015\}$

$\Rightarrow f^{-1} = f \Rightarrow (f \circ f^{-1} \circ f \circ f)(2x) = (f \circ f)(2x) = (f^{-1} \circ f)(2x) = 2x.$

CLASA a XI-a M1.

1. a) $D(x,y)=\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^3+1 & y^3+1 & z^3+1\end{vmatrix}=\begin{vmatrix}1 & 0 & 0\\ x & y-x & z-x\\ x^3+1 & y^3-x^3 & z^3-x^3\end{vmatrix}=(y-x)(z-x)\Big|\begin{matrix}C_3-C_2\\ =\end{matrix}$

$=\dots=(y-x)(z-x)(z-y)(x+y+z+2)=(y-x)(x-z)(y-z)(x+y+z+2)$

b) $D(2015^x,-3)=0\Rightarrow(-3-2015^x)(2015^x-2)(-1)\cdot(2015^x-1)=0\Rightarrow$

$\Rightarrow x=\log_{2015^2}2$; $x=0$.

c) $D(\log_{2015}x;-1)=0\Rightarrow(-1-\log_{2015}x)(\log_{2015}x-2)(-3)(\log_{2015}x+1)=0$

$\Rightarrow x=\frac{1}{2015}$; $x=2015^2$.

2. a) $X(a)=I_2+a\cdot\begin{pmatrix}2014 & -1007\\ -2 & 1\end{pmatrix}=I_2+a\cdot A$; $\det A=0\Rightarrow$

$\Rightarrow A^2=2015A$.

$X(a)\cdot X(b)=(I_2+aA)(I_2+bA)=I_2+aA+bA+abA^2=$

$=I_2+(a+b+2015ab)\cdot A$

b) $I_2=X(0)$; $X(a)\cdot X(a')=X(0)\Rightarrow a+a'+2015aa'=0$

$\Rightarrow a'(1+2015a)=-a\Rightarrow a'=\frac{-a}{1+2015a}$, $\forall a\neq-\frac{1}{2015}$.

c) $X^3(a)=X\left(2015^2\left(a+\frac{1}{2015}\right)^3-\frac{1}{2015}\right)\Rightarrow 2015^2\left(a+\frac{1}{2015}\right)^3=-\frac{1}{2015}$

$\Rightarrow a+\frac{1}{2015}=-\frac{1}{2015}\Rightarrow a=-\frac{2}{2015}$

3. $\lim\limits_{x\to\infty}\left(\sqrt[2014]{\quad}-\sqrt[2014]{\quad}\right)=\infty-\infty$.

$x=\frac{1}{y}$; $x\to\infty\Rightarrow y=0$.

$L=\lim\limits_{y\to0}\dfrac{\sqrt[2015]{1+2y+2y^{2015}}-\sqrt[2015]{1-2y^{2014}+y^{2015}}}{y}\overset{\frac{0}{0}}{=}\overset{l'H}{}$

$=\lim\limits_{y\to0}\left(\dfrac{2+2\cdot2015y^{2014}}{2015\sqrt[2015]{(1+2y+2y^{2015})^{2014}}}-\dfrac{-2\cdot2014y^{2013}+2015y^{2014}}{2015\cdot\sqrt[2015]{(\quad)^{2014}}}\right)=$

$=\frac{2}{2015}-0=\frac{2}{2015}$

4. $\lim\limits_{n\to\infty}\sqrt[n]{n}=1\Rightarrow L=(a+b)^0\in\mathbb{R}\Rightarrow a+b=1\Rightarrow b=1-a$.

$L=\lim\limits_{n\to\infty}\left(a\sqrt[n]{n}+1-a\right)^{\frac{n}{\ln n}}=\lim\limits_{n\to\infty}\left[\left(1+a(\sqrt[n]{n}-1)\right)^{\frac{1}{\sqrt[n]{n}-1}}\right]^{\frac{\sqrt[n]{n}-1}{\ln n}\cdot n}=$

$\lim\limits_{n\to\infty}\frac{\left(e^{\frac{1}{n}\ln n}-1\right)a}{\frac{1}{n}\ln n}$

$=e^{\lim\limits_{n\to\infty}\frac{\left(e^{\frac{1}{n}\ln n}-1\right)a}{\frac{1}{n}\ln n}}=e^a\in\mathbb{R}^*\Rightarrow a\in\mathbb{R}$.

CLASA a XI-a M2.

1. a) $D(x,y)=\begin{vmatrix} 1 & 0 & 0 \\ x+2 & y-x & 3-x \\ x^2+3 & y^2-x^2 & 9-x^2 \end{vmatrix} = (y-x)(3-x)\cdot\begin{vmatrix} 1 & 0 & 0 \\ x+2 & 1 & 1 \\ x^2+3 & y+x & 3+x \end{vmatrix} =$

$= (y-x)(3-x)\cdot\begin{vmatrix} 1 & 0 & 0 \\ x+2 & 1 & 0 \\ x^2+3 & y+x & 3-y \end{vmatrix} = (y-x)(3-x)(3-y) = (y-x)(x-3)(y-3)$.

b) $D(2015^x, 4)=0 \Rightarrow (2015^x-4)(2015^x-3)=0 \Rightarrow x=\log_{2015}4$; $x=\log_{2015}3$

c) $D(\log_{2015}x; -2)=0 \Rightarrow \left(-2-\log_{2015}x\right)\left(\log_{2015}x-3\right)(-2-3)=0 \Rightarrow$

$\Rightarrow \log_{2015}x=-2$ sau $\log_{2015}x=3 \Rightarrow x=2015^{-2}$ sau $x=2015^3$

2. a) $B\cdot A=\begin{pmatrix}1&0\\1&1\end{pmatrix}\cdot\begin{pmatrix}2&0\\3&2\end{pmatrix}=\begin{pmatrix}2&0\\5&2\end{pmatrix}$; $A\cdot B=\begin{pmatrix}2&0\\3&2\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}=\begin{pmatrix}2&0\\5&2\end{pmatrix}$

b) $X=\begin{pmatrix}x&y\\z&t\end{pmatrix}\in M_2(\mathbb{R})$ a.î. $XA=AX$.

$XA=\begin{pmatrix}x&y\\z&t\end{pmatrix}\begin{pmatrix}2&0\\3&2\end{pmatrix}=\begin{pmatrix}2x+3y&2y\\2z+3t&2t\end{pmatrix}$; $AX=\begin{pmatrix}2&0\\3&2\end{pmatrix}\begin{pmatrix}x&y\\z&t\end{pmatrix}=\begin{pmatrix}2x&2y\\3x+2z&3y+2t\end{pmatrix}$

$\Rightarrow y=0$; $x=t \Rightarrow X=\begin{pmatrix}x&0\\z&x\end{pmatrix}$

c) $x^2=A$; $x^3=x^2\cdot x=A\cdot X$; $x^3=x\cdot x^2=x\cdot A \Rightarrow X\cdot A=A\cdot X$

$\Rightarrow \exists\, x,y\in\mathbb{R}$ a.î. $X=\begin{pmatrix}x&0\\y&x\end{pmatrix}$

$X^2=\begin{pmatrix}x&0\\y&x\end{pmatrix}\begin{pmatrix}x&0\\y&x\end{pmatrix}=\begin{pmatrix}x^2&0\\2xy&x^2\end{pmatrix} \Rightarrow \begin{matrix}x^2=2\Rightarrow x=\pm\sqrt{2}\\ y^2=3\Rightarrow y=\pm\sqrt{3}\end{matrix} \Rightarrow$ 4 matrice

3. a) $f'(x)=\dfrac{x^2-4x-a}{(x-2)^2}$; $\begin{matrix}f'(x_0)=1\\ x_0=1\end{matrix}\Rightarrow \begin{matrix}f'(1)=1\Rightarrow -3-a=1\Rightarrow a=-2\\ f(1)=1\Rightarrow b=a\end{matrix}$

b) $f(x)=\dfrac{x^2-2}{x-2}$, $x\neq 2$.

$m=\lim\limits_{x\to\infty}\dfrac{x^2-2}{x^2-2x}=1\neq 0$; $n=\lim\limits_{x\to+\infty}\left(\dfrac{x^2-2}{x-2}-x\right)=\lim\limits_{x\to+\infty}\dfrac{2x-2}{x-2}=2\in\mathbb{R}$.

$\Rightarrow y=x+2$ as. oblică spre $\pm\infty$.

$\lim\limits_{\substack{x\to 2\\ x<2}} f(x)=\dfrac{2}{0_-}=-\infty$, $\lim\limits_{\substack{x\to 2\\ x>2}} f(x)=\dfrac{2}{0_+}=+\infty \Rightarrow x=2$ as. verticală.

f continuă pe $\mathbb{R}\setminus\{2\} \Rightarrow$ nu alte asimptote verticale.

c) $f'(x)=\dfrac{x^2-4x+2}{(x-2)^2}$; $f'(x)=0 \Rightarrow x^2-4x+2=0 \Rightarrow x_{1,2}=2\pm\sqrt{2}$.

x	$-\infty$		$2-\sqrt{2}$		2		$2+\sqrt{2}$		$+\infty$
$f'(x)$		$+$	0	$-$	$\nexists$	$-$	0	$+$	
$f(x)$	$-\infty$	↗	$f(x_1)$ max	↘	$-\infty$	↘	$f(x_2)$ min	↗	$+\infty$

f p.c. pe $(-\infty, 2-\sqrt{2}]$ și pe $[2+\sqrt{2}; \infty)$,
f p.d. pe $[2-\sqrt{2}; 2)$ și pe $(2; 2+\sqrt{2}]$.

4) a) $\lim\limits_{x\to\infty}\dfrac{e^x}{x+2}\overset{\infty}{=}\dots=\infty\notin\mathbb{R} \Rightarrow$ nu are as. spre $+\infty$; $m=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\dots=+\infty\notin\mathbb{R}$

$\lim\limits_{x\to-\infty}\dfrac{e^x}{x+2}=\dfrac{0}{\infty}=0 \Rightarrow y=0$ as. spre $-\infty$.

$\lim\limits_{\substack{x\to-2\\ x<-2}}\dfrac{e^x}{x+2}=-\infty$; $\lim\limits_{\substack{x\to-2\\ x>-2}} f(x)=+\infty \Rightarrow x=-2$ a.v. ; f cont. pe $\mathbb{R}\setminus\{-2\} \Rightarrow$ nu alte as. verticale.

b) $f'(x)=\dfrac{e^x(x+1)}{(x+2)^2}$; $f'(x)=0 \Rightarrow x=-1$;

x	$-\infty$		-2		-1		$+\infty$
$f'(x)$		$-$	$\nexists$	$-$	0	$+$	
$f(x)$	↘		$-\infty$	↘	$\frac{1}{e}$	↗	$+\infty$

f p.d. pe $(-\infty; -2)$ și pe $(-2; -1]$,
f p.c. pe $[-1; \infty)$

c) $f(0)\le f(x)\le f(1) \Rightarrow \frac{1}{2}\le f(x)<\frac{e}{3} \Rightarrow \frac{2}{e}\le f(x)\le 2$.

CLASA a XII-a IM1.

1. a) $x \in [0;1] \Rightarrow x^4 \le x^2 \le 1 \Rightarrow \frac{1}{x^2+1} \le \frac{1}{x^4+1} \le 1 \Rightarrow \frac{\pi}{4} \le \int_0^1 f(x)dx \le 1$

b) $\int_0^1 \left(\frac{f'(x)}{f(x)}\right)' dx = \frac{f'(x)}{f(x)}\Big|_0^1 = -8$; $f'(x) = \frac{-4x^3}{(x^4+1)^2}$

c) $f(x) = x + \sin x \Rightarrow f'(x) = 1+\cos x$, $f''(x) = -\sin x$.

$f(x)\cdot f''(x) - (f'(x))^2 = -x\sin x - 2 - 2\cos x$.

$F(x) \overset{DI}{=} \frac{f'(x)}{f(x)} + C = \frac{1+\cos x}{x+\sin x} + C$; $F(\frac{\pi}{2}) + C = \frac{2}{\pi+2} \Rightarrow \frac{1}{\frac{\pi}{2}+1} + C = \frac{2}{\pi+2}$

$\Rightarrow C = 0 \Rightarrow F(x) = \frac{1+\cos x}{x+\sin x}$

2. a) $I_1 = \int_0^1 \frac{\ln(x+1)}{x+1} dx$; $t = \ln(x+1) \Rightarrow dt = \frac{1}{x+1}dx$

$x = 0 \Rightarrow t = 0$
$x = 1 \Rightarrow t = \ln 2$ $\Rightarrow I_1 = \int_0^{\ln 2} t\, dt = \frac{1}{2}t^2\Big|_0^{\ln 2} = \frac{1}{2}\ln^2 2$.

b) $I_{n+1} - I_n = \int_0^1 \frac{1}{x+1} \ln \frac{x^{n+1}+1}{x^n+1} dx$

$0 \le x^{n+1}+1 \le x^n+1 \Rightarrow \ln \frac{x^{n+1}+1}{x^n+1} \le 0 \Rightarrow (I_n)_n$ m. d.

c) $\ln(1+x) \le x$, $\forall x > 0$ (demo.) $\Rightarrow \frac{\ln(1+x^n)}{x+1} \le \frac{x^n}{x+1} \le x^n$, $\forall x \in [0,1]$

$\Rightarrow \ln(1+x^n) \le x^n$, $\forall x > 0 \Rightarrow 0 \le I_n \le \frac{1}{n+1} \to 0 \Rightarrow \lim I_n = 0$.

3. a) $\chi(a) = I_2 + a\cdot A$; $\det A = 0 \Rightarrow A^2 = 2015A$

$\chi(a)\cdot\chi(b) = (I_2 + aA)(I_2 + bA) = I_2 + aA + bA + ab A^2 =$

$= I_2 + (a+b+2015ab)A$

$x \ne -\frac{1}{2015}$, $y \ne -\frac{1}{2015} \Rightarrow \left(x+\frac{1}{2015}\right)\left(y+\frac{1}{2015}\right) \ne 0 \Rightarrow$

$\Rightarrow 2015\left(x+\frac{1}{2015}\right)\left(y+\frac{1}{2015}\right) - \frac{1}{2015} \ne -\frac{1}{2015}$.

b) $I_2 = \chi(0) \in G$; $\chi(a)$ inv. $(\Leftrightarrow) \exists \chi(a') \in G$ a.i. $\chi(a)\cdot\chi(a') = \chi(a')\cdot\chi(a) = \chi(0) \Rightarrow a' = \frac{-a}{1+2015a} \in \mathbb{R}\setminus\{-\frac{1}{2015}\}$

c) $\chi^5(a) = \chi\left(-\frac{2}{2015}\right) \Rightarrow 2015^4\left(a+\frac{1}{2015}\right)^5 - \frac{1}{2015} = -\frac{2}{2015} \Rightarrow$

$\Rightarrow a + \frac{1}{2015} = -\frac{1}{2015} \Rightarrow a = \frac{-2}{2015} \in \mathbb{R}\setminus\{-\frac{1}{2015}\}$.

4. a) $x+\hat{2} \mid f \Rightarrow f(\hat{3}) = \hat{0} \Rightarrow a = \hat{4}$

b) $f = \hat{2}x^4 + \hat{2}x^3 + x^2 + \hat{2}x + \hat{4} = (x-\hat{2})(x-\hat{3})(\hat{2}x^2+\hat{2}x+\hat{3})$

$g = \hat{2}x^2+\hat{2}x+\hat{3}$ ireductibil (pentru $p=3$, $\mathbb{Z}_5$ corp, g nu are rădăcini în $\mathbb{Z}_5$).

c) $\hat{3}\cdot(a+\hat{1})^2 = \hat{2} \Rightarrow (a+\hat{1})^2 = \hat{4} \Rightarrow$ $a+\hat{1} = \hat{2} \Rightarrow a = \hat{1}$

$a+\hat{1} = \hat{3} \Rightarrow a = \hat{2}$.

CLASA a XII-a M2.

1. a) $x*y = 2015\left(x+\frac{1}{2015}\right)\left(y+\frac{1}{2015}\right) - \frac{1}{2015}$.

Alg $x = \frac{2}{2015} \in \mathbb{Q}\setminus\mathbb{Z}$, $y = \frac{-2}{3\cdot 2015} \in \mathbb{Q}\setminus\mathbb{Z} \Rightarrow x*y = 0 \in \mathbb{Z}$

b) $(x*y)*z = \dots = 2015^2\left(x+\frac{1}{2015}\right)^3 - \frac{1}{2015}$

$x*(y*z) = \dots = 2015^2\left(x+\frac{1}{2015}\right)^3 - \frac{1}{2015}$ $\Rightarrow$ asociativitate

c) $x*x*x = -\frac{3}{2015} \Rightarrow 2015^2\left(x+\frac{1}{2015}\right)^3 = -\frac{1}{2015} \Rightarrow \left(x+\frac{1}{2015}\right)^3 = -\frac{1}{2015^3} \Rightarrow$

$\Rightarrow x+\frac{1}{2015} = -\frac{1}{2015} \Rightarrow x = \frac{-2}{2015} \in \mathbb{R}$.

2. a) $f(\hat{3}) = \hat{4}+\hat{4}+\hat{3}a+\hat{1} = \hat{4}+\hat{3}a$; $3\hat{a}+\hat{4} = \hat{2} \Rightarrow \hat{3}a = \hat{3} \Rightarrow a = \hat{1}$ sol. unica.

b) $f = \hat{2}x^3+\hat{4}x^2+\hat{3}a+\hat{4} = \hat{2}(x-\hat{2})^2(x-\hat{4})$

c) $f(\hat{3}) = \hat{4}+3\hat{a}$; $\hat{3}(\hat{4}+\hat{3}a) = \hat{2} \Rightarrow \hat{4}+\hat{3}a = \hat{4} \Rightarrow \hat{3}a = \hat{0} \Rightarrow a = \hat{0}$ sol (!)

3. a) F derivabila pe $(0;+\infty)$

$F'(x) = e^x + 1 - \frac{1}{x} = f(x), \forall x>0 \Rightarrow F$ o primitiva a lui f

b) $\int_1^2 x\cdot e^{2015x}\,dx = \frac{1}{2015}e^{2015x}\cdot x\Big|_1^2 - \frac{1}{2015^2}e^{2015x}\Big|_1^2 =$

$= \frac{2\cdot e^{2015\cdot 2} - e^{2015}}{2015} - \frac{e^{4030}-e^{2015}}{2015^2} = \dots$

c) $\int_1^e \left(e^x+1-\frac{1}{x}\right)dx = \left(e^x+x-\ln x\right)\Big|_1^e = e^e - 2$

$e^e - 2 = e^m - 2 \Rightarrow m = e$.

4. a) $\int_0^1 g(x)\,dx = f(x)\Big|_0^1 = 1-0 = 1$.

b) $G(x) = \frac{x^3}{x+1} + C$;

$G(1) = \frac{1}{2} \Rightarrow C = 0 \Rightarrow G(x) = \frac{x^3}{x+1}$, $\forall x \in [0;1]$

c) $0 \le x \le 1 \Rightarrow 1 \le x+1 \le 2 \Rightarrow \dots \Rightarrow \left(\frac{x^3}{2}\right)^2 \le \left(\frac{x^3}{x+1}\right)^2 \le (x^3)^2$, $\forall x \in [0;1]$

$\Rightarrow \frac{1}{4}\cdot\frac{1}{7}\cdot U \le V \le U\cdot\frac{1}{7} \Rightarrow \frac{U}{20} \le V \le \frac{U}{7}$.

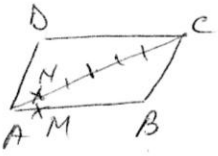
CLASA a IX-a M2.

1. $f(f(0)) = f(b-2) = (a-1)(b-2) + b - 2$
$f(2) = 2a - 2 + b - 2 = 2a + b - 4$ și $f(2) = b \Rightarrow 2a = 4 \Rightarrow a = 2$.
$f(f(0)) = f(3) \Rightarrow 2(b-2) = 3 + b - 2 \Rightarrow b = 5$

2. $\div a, b, c \Rightarrow 2b = a + c$
a) $f(1) = 15 \Rightarrow a + b + c = 15$ / $\Rightarrow b = 5 \Rightarrow a + c = 10$ / $\Rightarrow c = 2, a = 8$.
$a + c = 2b$ $\quad a = 4c$.

b) $f(x) = 8x^2 + 5x + 2$.
$a = 8 > 0 \Rightarrow \text{Im} f = \left[-\frac{\Delta}{4a}; +\infty \right) = \left[\frac{39}{32}; +\infty \right)$

3. a) $\frac{AN}{AC} = \frac{1}{6} \Rightarrow \frac{AN}{NC} = \frac{1}{5}$



$\vec{DN} = \frac{1}{\frac{1}{5}+1} \cdot \vec{DA} + \frac{1}{6} \cdot \vec{DC} = \frac{5}{6}\vec{DA} + \frac{1}{6}\vec{DC}$

b) M, N, D coliniare $\Rightarrow \vec{DN} \parallel \vec{DM}$
$\vec{AM} = \alpha \cdot \vec{AB} \Rightarrow \vec{AM} = \alpha(\vec{AM} + \vec{MB}) \Rightarrow (1-\alpha)\vec{AM} = \alpha \vec{MB}$
$\vec{DM} = \frac{1}{1 + \frac{\alpha}{1-\alpha}} \cdot \vec{DA} + \frac{\alpha}{1} \vec{DB} = \alpha \vec{DB} + (1-\alpha)\vec{DA} =$
$= \alpha \cdot (\vec{DC} + \vec{CB}) + (1-\alpha)\vec{DA} = \alpha \vec{DC} + \vec{DA}$
$\vec{DN} \parallel \vec{DM} \Rightarrow \frac{5}{6} = \frac{1}{6\alpha} \Rightarrow \alpha = \frac{1}{5}$

4. a) $\vec{OA} = -2\vec{i} + 3\vec{j}$
b) $M(a; b)$; $a, b \in \mathbb{R}$.
$2(a+2)\cdot\vec{i} + 2(b-3)\cdot\vec{j} = 3 \cdot (4\vec{i} - 2\vec{j}) \Rightarrow \begin{matrix} 2a+4 = 12 \Rightarrow a = 4 \\ 2b - 6 = -6 \Rightarrow b = 0 \end{matrix}$
$\Rightarrow M(4; 0)$
c) $G\left(\frac{-2-1+3}{3}, \frac{3+4+2}{3} \right) \Rightarrow G(0; 3)$.
$\vec{AG} = -2\vec{i} \Rightarrow AG = 2$.